Exercise 329

For the following problems, determine the largest domain on which the function is one-to-one and find the inverse on that domain.

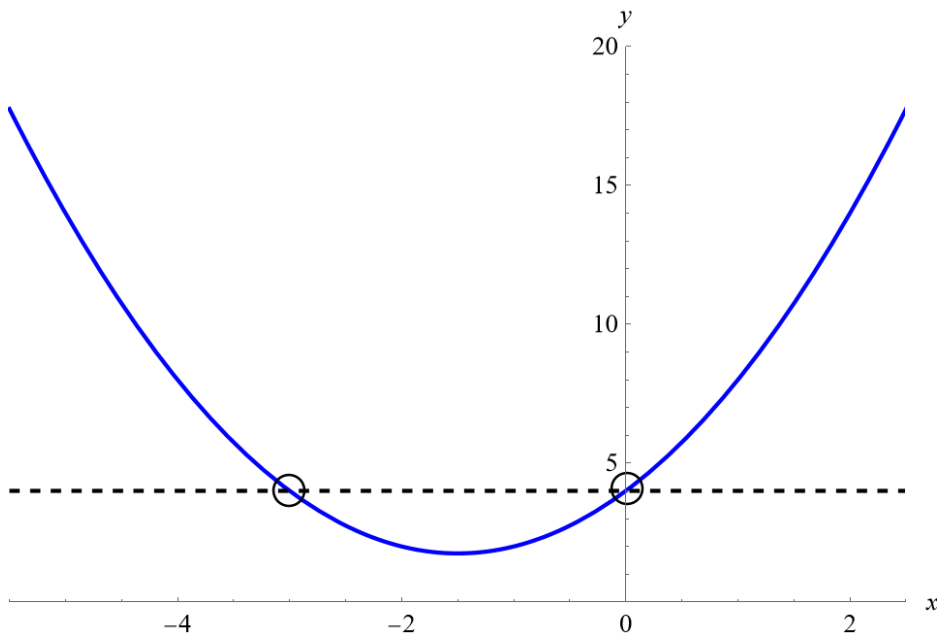
$$f(x) = x^2 + 3x + 4$$

Solution

$f(x)$ is a quadratic function, so the graph is of a parabola. To graph it exactly, complete the square.

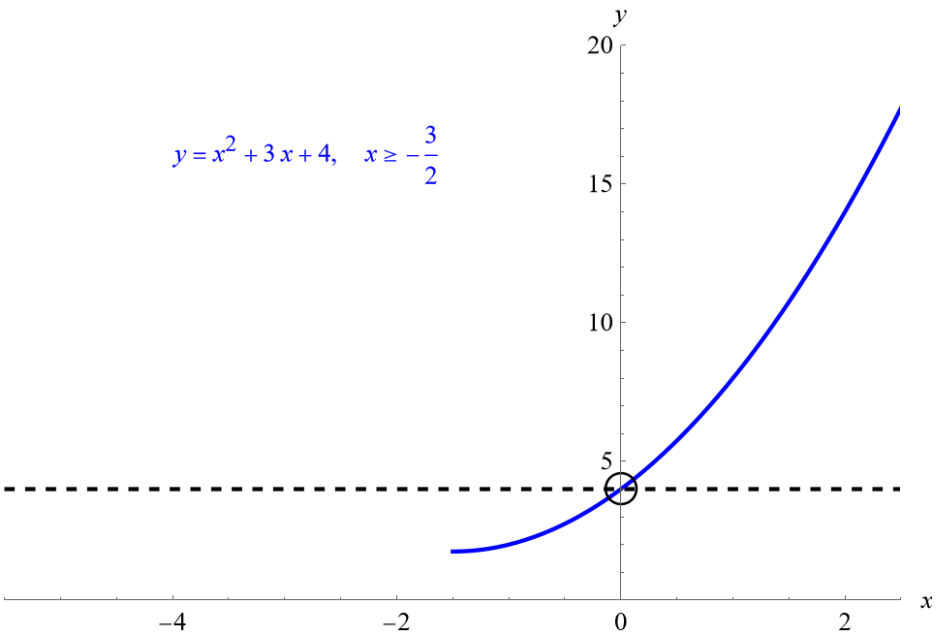
$$\begin{aligned} f(x) &= \left(x^2 + 3x + \frac{9}{4}\right) - \frac{9}{4} + 4 \\ &= \left(x + \frac{3}{2}\right)^2 + \frac{7}{4} \end{aligned}$$

This parabola is shifted upward by 7/4 units and shifted to the left by 3/2 units as shown below.



It fails the horizontal line test, so it's not one-to-one and therefore does not have an inverse.

However, by taking the restriction of $f(x)$ to $x \geq -3/2$, it passes the horizontal line test and does have an inverse.



Replace x with y, and replace $f(x)$ with x in the equation.

$$x = y^2 + 3y + 4$$

Solve for y.

$$y^2 + 3y + (4 - x) = 0$$

$$y = \frac{-3 \pm \sqrt{9 - 4(4 - x)}}{2}$$

We choose the plus sign because the inverse function has to be the mirror image of the function over the line $y = x$.

$$y = \frac{-3 + \sqrt{9 - 4(4 - x)}}{2}$$

Simplify the right side.

$$y = \frac{-3 + \sqrt{4x - 7}}{2}$$

$f(x)$ and $f^{-1}(x)$ are shown in the graph below.